Journal of Applied Mechanics and Technical Physics, Vol. 38, No. 3, 1997

TAKING INTO ACCOUNT THE DYNAMICS IN DESCRIPTION OF FRACTURE OF BRITTLE MEDIA BY AN EXPLOSION OF A CORD CHARGE

E. N. Sher

UDC 622.235

This study is a continuation of [1, 2] in which the parameters of explosive fracture caused by a deep-hole charge in brittle rocks were estimated. In [1], such estimates were obtained in a static approximation. The static approach does not allow one to estimate the time parameters of the process and to make allowance for the effect of flowing of gaseous detonation products over a well and tamping. For a dynamic description of brittle-medium fracture by an explosion of a cord charge, it is important to take into account the dynamics of evolution of the radial-crack zone. In [2], the author proposed an energy approach to the description of such a zone within the framework of the zone theory of explosion in solid media [3–6]. Near the HE charge, this zone is usually divided into a grinding zone, a radial-crack zone, and an external elasticity zone. Although it is possible to describe the dynamics of evolution of the zones in question by numerical methods, it is of interest to take into account approximately the dynamics of the process and to estimate the parameters computationally and analytically.

In the present study, we apply an exact dynamic description to the grinding zone in which inertial forces are maximal. For the radial-crack and elasticity zones, a quasi-static description is used to determine stresses and strains. In this case, we use the dynamics of the process in determining the radial-crack front by the dynamic crack strength of a medium.

Formulation of the Problem. We shall consider the axisymmetric deformation and fracture of an isotropic brittle medium under the action of an explosion of an infinite cylindrical HE charge. Charge detonation is assumed to be instantaneous, and deformation of a medium is assumed to be plane. Consideration as a whole is performed without taking into account the waves in the isochronous model of Mashukov et al. [7] in which the shock wave in a medium is assumed to leave rapidly the explosion cavity, carrying away a certain portion of energy, the main events are assumed to occur after it, and the stress and strain fields are assumed to be close to static fields determined for a load at a given moment of time. One can then separate several stages in the evolution of the explosion.

(1) The wave of fracture moves with a velocity exceeding a maximum crack velocity $v_{\max}$. At this stage, there are two zones: a plastic zone for $a \leqslant r \leqslant b$ and an elastic one for $r \geqslant b$, where $a(t)$ is the explosion-cavity radius, $b(t)$ is the boundary of the plastic zone, and r is the current radius.

(2) When the rate of growth of the plastic zone decreases and the inequality $\dot{b} \leqslant v_{\max}$ is satisfied, a radial-crack zone appears for $b \leqslant r \leqslant l$ [$l(t)$ is the radius of the radial-crack zone front]. An elasticity zone appears for $r \geqslant l(t)$.

For stresses and strains in the radial-crack zone, the static relations for loads at the current moment are also assumed to be satisfied. The crack density in this zone is an additional parameter of the process. Analysis of the experimental data has shown that in the process of development of the radial-crack zone, the number of radial cracks decreases. Some cracks stop, thus giving the others the possibility of further development. To estimate the crack density at various distances from the explosion center, we performed calculations for the second stage for some values of the number N of cracks, for example, $N = 128$, 64, 32, 16, 8, and 4.

Mining Institute, Siberian Division, Russian Academy of Sciences, Novosibirsk 630091. Translated from Prikladnaya Mekhanika i Tekhnicheskaya Fizika, Vol. 38, No. 3, pp. 174–183, May–June, 1997. Original article submitted November 30, 1995; revision submitted January 18, 1996.

Computational Model. Let us write the equations of motion and the general solutions for each zone.

The gas pressure in the explosion cavity is calculated using the modified Jones–Miller adiabat [6] for a trotyl cylindrical charge in the form

$$p(a) = \begin{cases} p_0(a/a_0)^{-2\gamma_1}, & a \leqslant a^*, \\ p_0(a^*/a_0)^{-2\gamma_1}(a/a^*)^{-2\gamma_2}, & a \geqslant a^*, \end{cases}$$

where $p_0 = 3.32 \cdot 10^9$ Pa, $\gamma_1 = 3$, $\gamma_2 = 1.27$, $a^*/a_0 = 1.89$, a_0 is the initial charge radius, and a is the cavity radius.

It is assumed that the equations of motion of an incompressible loose medium is satisfied in the grinding zone near the charge. For the one-dimensional case of axial symmetry, in a cylindrical coordinate system (r, ϑ) we have

$$\rho\Big(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial r}\Big) = \frac{\partial \sigma_r}{\partial r} + \frac{\sigma_r - \sigma_\vartheta}{r}. \tag{1}$$

Here ρ is the density of the medium, v is the radial velocity, and σ_r and σ_ϑ are the stress-tensor components, respectively. The Coulomb law [6] $\tau = C - \sigma \tan\varphi$ (τ and σ are the tangential and normal stresses on a shear site) is used as a constitutive relation. In expansion of the explosion cavity in this zone, in terms of the principal stresses of the axisymmetric problem we have

$$(1+\alpha)\sigma_\vartheta - \sigma_r - Y = 0 \qquad [Y = 2C\cos\varphi/(1-\sin\varphi), \quad \alpha = 2\sin\varphi/(1-\sin\varphi)]. \tag{2}$$

In compression of the cavity, the yield condition takes the form $(1+\alpha)\sigma_r - \sigma_\vartheta - Y = 0$ which can formally be reduced to (2) with new α_1 and Y_1: $(1+\alpha_1)\sigma_\vartheta - \sigma_r - Y_1 = 0$, $\alpha_1 = -\alpha/(1+\alpha)$, and $Y_1 = -Y/(1+\alpha)$. Relation (2) is satisfied provided that the shear sites are parallel to the axis of symmetry. This is true if the initial compression by rock pressure along the z axis is the average pressure relative to the other axes.

The incompressibility condition in the grinding zone enables one to write the velocity of any point of the medium as

$$v(r,t) = \dot{a}a/r, \tag{3}$$

where $a(t)$ is the radius of the explosion cavity.

Excluding σ_ϑ from (1) by means of (2) and using (3), for σ_r we obtain the equation

$$\frac{\partial \sigma_r}{\partial r} + \frac{\alpha}{(1+\alpha)}\frac{\sigma_r}{r} = \frac{Y}{(1+\alpha)r} + \rho\left(\frac{\ddot{a}a + \dot{a}^2}{r} - \frac{(\dot{a}a)^2}{r^3}\right)$$

whose general solution is representable in the form

$$\sigma_r = \frac{Y}{\alpha} + \rho\left((\ddot{a}a + \dot{a}^2)\frac{1+\alpha}{\alpha} + (\dot{a}a)^2\frac{(1+\alpha)}{(2+\alpha)}\frac{1}{r^2}\right) + F(t)r^{-\alpha/(1+\alpha)}. \tag{4}$$

For the displacement u_b, at the boundary of the grinding zone $r = b(t)$, from the incompressibility condition it follows that

$$a^2 - a_0^2 = b^2 - (b - u_b)^2. \tag{5}$$

For $b \leqslant r \leqslant l$, in the radial-crack zone we have

$$\begin{gathered} \sigma_\vartheta = 0, \qquad \sigma_r = -P_b b/r, \\ \frac{du}{dr} = -(1-\nu^2)P_b b/(Er), \qquad u = u_0(t) - (1-\nu^2)(P_b/E)b\ln(r/a_0), \end{gathered} \tag{6}$$

where P_b is the radial pressure for $r = b$, u is the radial displacement, E and ν are the Young and Poisson moduli of the fractured medium, and $u_0(t)$ is an arbitrary function of time.

In plane deformation, in the elasticity zone, for $r \geqslant l(t)$ we have the general static solution

$$\sigma_r = -P - EB/[(1+\nu)r^2], \qquad \sigma_\vartheta = -P + EB/[(1+\nu)r^2], \tag{7}$$

$$u = B/r - (1+\nu)(1-2\nu)rP/E.$$

Here P is the rock pressure in the medium and B is an arbitrary constant which is a function of time in the quasi-static consideration.

The arbitrary functions $F(t)$, $u_0(t)$, and $B(t)$ in (4), (6), and (7) are defined by the initial and boundary conditions.

At the first stage of development of the grinding zone, the following boundary conditions are satisfied:
— at the explosion cavity, the radial stress in the grinding zone is equal to the gas pressure at the cavity:

$$\sigma_r = -p(a) \quad \text{for} \quad r = a(t); \tag{8}$$

— in front of the fragmentation wave, in the elastic medium the shear-fracture Coulomb–Moore criterion holds:

$$(1+\alpha_2)\sigma_\vartheta - \sigma_r - Y_2 = 0 \quad \text{for} \quad r = b(t), \tag{9}$$

— and displacements are continuous:

$$u(b-0) = u(b+0) = u_b. \tag{10}$$

The fracture criterion (9) corresponds to the rectilinear envelope of the Moore circles and was confirmed for rocks by Mashukov et al. [7]. The parameters α_2 and Y_2 can be found from the uniaxial tension σ_t and compression σ_c tests:

$$\alpha_2 = \sigma_c/\sigma_t - 1, \qquad Y_2 = \sigma_c. \tag{11}$$

As an illustration, we shall consider Plexiglas with $\sigma_c = 1.6 \cdot 10^8$ Pa and $\sigma_t = 6 \cdot 10^7$ Pa, according to static tests. Using formulas (11), we find $\alpha_2 = 1.66$ and $Y_2 = 1.6 \cdot 10^8$ Pa. For rocks from the 6th to the 20th strength category, according to (11), from static uniaxial tests we have $\alpha_2 = 7$–12 and $Y_2 = (0.6$–$2.7) \cdot 10^8$ Pa. The difference of the coefficients of the elastic-medium fracture criterion (9) and the medium-yield condition (2) in the grinding zone is considerable as compared with those in [3, 6]. The similarity of Eqs. (2) and (9) simplifies calculations but is not obligatory; for example, as (9), we could use the dependence $\sigma_1 - \sigma_2 = f(\sigma_1/\sigma_2)$ (σ_1 and σ_2 are the principal stresses), which is determined for many rocks in [8].

In a quasi-static approach, stresses are usually assumed to be continuous [6] in passage through the fragmentation wave. The kinetic energy of the medium in the grinding zone increases with increasing b owing to the attachment of the moving layers of the elastic zone which has no kinetic energy from a formal point of view. This paradox is due to the fact that inertial terms are ignored in the description of the elastic zone. As a correction to the quasi-static description, we assume that the particles of the medium accelerate in the fragmentation wave from zero velocity to $\dot{u}_b$. For the stress in the grinding zone, with $r = b(t) - 0$, this yields the following equation:

$$\sigma_r(b-0) = -\rho\dot{u}_b\dot{b} + \sigma_r(b+0). \tag{12}$$

For $(a^2 - a_0^2)/b^2 \ll 1$, from conditions (5), (9), and (10) we find

$$\frac{b}{a} = n\sqrt{1 - \frac{a_0^2}{a^2}}, \qquad n = \sqrt{\frac{E(2+\alpha_2)}{2(1+\nu)(Y_2 + P\alpha_2)}}, \qquad \frac{u_b}{b} = \frac{1}{2n^2},$$
$$\sigma_r(b+0) = -[Y_2 + 2P(1+\alpha_2)]/(2+\alpha_2). \tag{13}$$

It follows from (12) and (13) that

$$\sigma_r(b-0) = \sigma_r(b+0) - \rho a^2\dot{a}^2/[2(a^2 - a_0^2)]. \tag{14}$$

In what follows, we use nondimensional variables. We use a_0 as a length scale, a_0/c_0 as time ($c_0^2 = E/\rho$), and E as a stress. Substituting the general solution (4) into the boundary conditions (8) and (14) and using (13), for $a(t)$ we obtain the following camouflet equation in nondimensional form:

$$K_1(a)a\ddot{a} + (K_1(a) - K_2(a))\dot{a}^2 + K_3 - p(a) = 0. \tag{15}$$

Here $K_1 = ((1+\alpha)/\alpha)[m^{\alpha/(1+\alpha)} - 1]$, $K_2 = ((1+\alpha)/(2+\alpha))[1 - m^{-(2+\alpha)/(1+\alpha)}] - (a^2/2(a^2-1))m^{\alpha/(1+\alpha)}$, $K_3 = ((Y_2 + 2(1+\alpha_2)P)/(2+\alpha_2) + Y/\alpha)m^{\alpha/(1+\alpha)} - Y/\alpha$, and $m = n\sqrt{1 - 1/a^2} = b/a$. Because of the approximate knowledge of $b(a)$ in (13), we take the initial conditions of the problem for $t = 0$ and $a = b = 1$ at the displaced point at $t = 0$ in the form

$$a = \sqrt{\frac{n^2}{n^2-1}} + \varepsilon, \quad \dot{a} = 0. \tag{16}$$

For (16), with $\varepsilon \ll 1$, from (13) it follows that $b \approx \sqrt{n^2/(n^2-1)} + n^2\varepsilon$.

The difference of the initial values of a and b from unity in (16) is negligible if $n \gg 1$. Here $a \approx b \approx 1 + 1/(2n^2)$. For rocks, $n \approx 10$, and, for $n = 7$, such a deviation amounts to 0.01. Calculations performed using Eq. (15) and the initial condition (16) have shown that as ε decreases, the integral curves approach the limiting shape. It was further assumed that $\varepsilon = 0.02$.

From these calculations, it follows that at the beginning of the process of fracture development the propagation velocity of the fragmentation zone grows intensely, reaches a maximum, and then diminishes. In decreasing this velocity to the maximum crack velocity in this medium, the character of fracture propagation can change. If the stress state allows a great deal of radial cracks to move away from the fracture front (larger than N_1), then propagating with a higher (compared with the fragmentation front) velocity, these cracks will cut the material before the fragmentation front into radial rods, make it free from tension in the tangential direction and thereby strengthening it against fragmentation in the front. In this case, the fragmentation front will stop, and fracture will occur owing to the propagation of the radial-crack zone. If the load-consistent number of cracks which have the front velocity is not sufficient for unloading of the tangential tension before the fragmentation front ($N < N_1$), the front will propagate according to (15) until N is equal to N_1. After that, the fragmentation front and the radial-crack zone stop to propagate. Introduction of the threshold value N_1 is a simplifying hypothesis of the proposed calculational scheme. In calculations, $N_1 = 64$.

For the second stage of fracture in which three zones exist, an equation of the type of (15) is derived from the boundary conditions in the explosion cavities and at the boundaries of the zones.

In the explosion zone, $\sigma_r = -p(a)$ for $r = a(t)$; at the boundary of the grinding zone, we have

$$u(b-0) = u(b+0), \quad \sigma_r(b-0) = \sigma_r(b+0) = -\sigma_1, \quad \sigma_1 \leqslant \sigma_c \tag{17}$$

for $r = b(t)$, and in the front of radial cracks, $u(l-0) = u(l+0)$ and $\sigma_r(l-0) = \sigma_r(l+0) = q$ for $r = l(t)$. With satisfaction of the equality in the stress condition (17), fragmentation of the radial cracks in the rod zone occurs, and $b(t)$ increases with time. In the opposite case $[\sigma_r(b+0) > -\sigma_c]$, the boundary of the grinding zone is immobile ($\dot{b} = 0$).

Using the solutions (5)–(7) and excluding $u_0(t)$ from (17), in the approximation $(a^2-1)/b^2 \ll 1$, we have

$$u_b \approx (a^2-1)/(2b), \ (a^2-1)/(2b^2) + (1+\nu)(Pl/b - \sigma_1) + (1-\nu^2)\sigma_1 \ln(b/l) - (1+\nu)(1-2\nu)P(1 - l/b) = 0. \tag{18}$$

For $\sigma_1 = \sigma_c$, differentiating with respect to t, from (18) we find

$$\dot{b} = \frac{\dot{l}((1-\nu^2)\sigma_c(b/l) - 2(1-\nu^2)P) - \dot{a}(a/b)}{(1-\nu^2)\sigma_c - (a^2-1)/b^2 - 2(1-\nu^2)P(l/b)} \quad \text{for} \quad \sigma_1 = \sigma_c,$$
$$\dot{b} = 0 \qquad \text{for} \quad \sigma_1 < \sigma_c. \tag{19}$$

Substituting the solution (4) into the boundary conditions in the explosion cavity and at the boundary of the grinding zone and excluding $F(t)$, we obtain the following camouflet equation:

$$\overline{K_1}a\ddot{a} + (\overline{K_1} - \overline{K_2})\dot{a}^2 + \overline{K_3} - p(a) = 0. \tag{20}$$

Here

$$\overline{K_1} = \frac{1+\alpha}{\alpha}\Big(\Big(\frac{b}{a}\Big)^{\alpha/(1+\alpha)} - 1\Big); \qquad \overline{K_2} = \frac{1+\alpha}{2+\alpha}\Big(1 - \Big(\frac{b}{a}\Big)^{-(2+\alpha)/(1+\alpha)}\Big);$$

$$\overline{K_3} = \left(\sigma_1 + \frac{Y}{\alpha}\right)\left(\left(\frac{b}{a}\right)^{\alpha/(1+\alpha)} - 1\right) + \sigma_1;$$

and, for $\sigma_1 < \sigma_c$, we obtain

$$\sigma_1 = \frac{(a^2-1)/(2b^2) + (1+\nu)P(2(1-\nu)l/b - 1 + 2\nu)}{(1+\nu) + (1-\nu^2)\ln(l/b)}.$$

For $l(t)$, in dimensionless form, we have

$$\dot{l} = \begin{cases} \dfrac{v_{\max}}{c_0}\,\dfrac{1-\exp\left(-\beta(\sqrt{\gamma/\gamma_0}-1)\right)}{1-\exp\left(-\beta(\sqrt{\gamma_1/\gamma_0}-1)\right)}, & \gamma_0 < \gamma < \gamma_1, \\ \dfrac{v_{\max}}{c_0}, & \gamma \geqslant \gamma_1, \end{cases} \tag{21}$$

where γ_0 and γ_1 are the specific surface energy of fracturing at the beginning of displacement and branching, respectively.

The dependence (21) was proposed by the author in [9] as an interpolation dependence for experimentally determinated certificate dependences of a number of brittle media that relate yielding and the crack propagation velocity.

The current value of γ is found from the energy condition in the radial-crack front [2]. In plane deformation and axial symmetry, we have

$$2\gamma N/(2\pi l) = 0.5(\sigma_r^{\text{elast}}\varepsilon_r^{\text{elast}} + \sigma_\vartheta^{\text{elast}}\varepsilon_\vartheta^{\text{elast}}) - 0.5\varepsilon_r^{\text{st}}\sigma_r^{\text{st}} + \sigma_r^{\text{st}}(\varepsilon_r^{\text{st}} - \varepsilon_r^{\text{elast}}).$$

Here σ_r^{elast}, $\sigma_\vartheta^{\text{elast}}$, $\varepsilon_r^{\text{elast}}$, and $\varepsilon_\vartheta^{\text{elast}}$ are the stresses and strains before the crack-zone front in the elastic zone for $r = l + 0$ and σ_r^{st} and $\varepsilon_r^{\text{st}}$ are the stresses and strains in the rod of radial cracks for $r = l - 0$. Substitution of the stresses and strains from (6) and (7) yields

$$2\gamma = \frac{1-\nu^2}{E}(2P+q)^2\frac{\pi l}{N}; \qquad q = -\sigma_1\frac{b}{l}. \tag{22}$$

The dependence (22) agrees well with the solutions of the problems of the plane elasticity theory on equilibrium of a star of N cracks in expansion by the inner pressure σ_1 over the radius b and by the rock pressure P. Hence, from (22) it follows that $K_{\text{I}} = 2P\sqrt{\pi l/N} - \sigma_1 b\sqrt{\pi/Nl}$ which coincides, for large S, with the asymptotic relations of the solutions of the aforementioned problems [10, 11].

Calculations of the Dynamics of a Fragmentation Wave. The dynamics of the explosion-cavity radius was calculated by Eq. (15) which was derived for the case of fragmentation-wave propagation in an elastic medium until the full stoppage of propagation of the cavity and of the fragmentation wave. The values obtained for the cavity and fragmentation-wave radii at the moment when the radial-crack zone front separates from the fragmentation-wave front were used as initial data for calculations of the second stage of fracture of brittle media in the radial-crack zone.

The characteristic explosion-cavity trajectories in a and t coordinates are given in Fig.1. In the calculations, we used the following parameters: $Y = 8 \cdot 10^5$ Pa and $\alpha = 0.1$, 0.5, and 1.0 (curves 1–3). The remaining parameters were taken from the initial set of parameters, which determine their average values for rocks: $E = 10^{10}$ Pa, $\rho = 2500$ kg/m^3, $\nu = 0.5$, $Y = 8 \cdot 10^7$ Pa, $\alpha = 4$, $\sigma_c = 8 \cdot 10^8$ Pa, $\gamma_0 = 150$ J/m^2, $\gamma_1 = 1500$ J/m^2, $v_{\max} = 650$ m/sec, $\beta = 1$, $a_0 = 0.0225$ m, $\alpha_2 = 11.2$, $P = 10^5$ Pa, $p_0 = 3.32 \cdot 10^9$ Pa, and $Y_2 = 8 \cdot 10^8$ Pa. Curve 4 in Fig. 1 shows the dependence $a(t)$ for the basic variant. Clearly, owing to the internal friction of the medium in the grinding zone, there are aperiodic and oscillatory regimes of explosion-cavity motion. Note that the inverse motion occurs only for fairly small values of α and of the friction angle [$\varphi < 20°$ by (2)]. In the opposite case, in development of the grinding zone, the ground material would be dynamically thrown, in the expansion phase, deep in the medium and would not return after stoppage to the explosion center, thus remaining blocked in the compression phase of the bulk material. Moreover, the elastic component of compression which is presumably small as compared with the plastic component was not taken into account in calculations. This blocking turns out also to be determining for the scale of fracture in

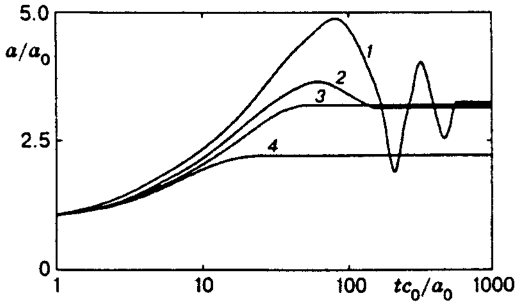

Fig. 1

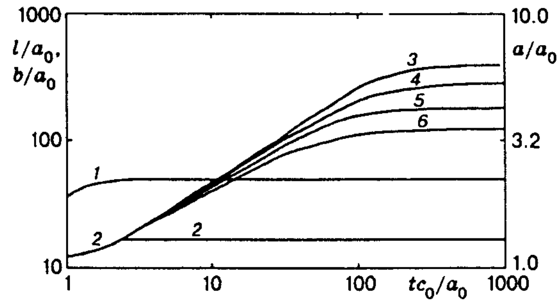

Fig. 2

the radial-crack zone, because it causes loading of a brittle medium which is close to a static loading by an expanded piston behind the front of the stopped fragmentation wave.

Static Estimation of Fracture in the Radial-Crack Zone According to Data on the Medium's Expansion in a Fragmentation Wave. If after the fragmentation front and the explosion cavity stop to propagate, the values of the cavity radius and of the fragmentation-wave front are a_k and b_k and the back collapse of the cavity is impossible because of friction in the grinding zone, one can find, by formula (5) or by an approximate formula for $a^2 - a_0^2 \ll b^2$, the displacement u_b at the inner boundary of the radial-crack zone. Let us calculate the limiting number of cracks N in the radial-crack zone of length l. Using (20) and (22), we obtain

$$N = \frac{\pi l(1-\nu^2)}{2\gamma_0}[q+2P]^2, \tag{23}$$

where

$$q = -\frac{1}{l}\frac{u_b + (1+\nu)lP}{(1+\nu)+(1-\nu^2)\ln(l/b_k)}; \qquad u_b \approx \frac{a_k^2 - 1}{2b_k}.$$

For large l/b_k and small P, we have in dimensional form

$$N \approx \frac{\pi E(a_k^2 - a_0^2)^2}{8\gamma_0 l b_k^2 (1-\nu^2)\ln^2(l/b_k)}. \tag{24}$$

Calculations of the Dynamics of Fracture of a Brittle Medium by an Explosion of a Cylindrical Charge in Fragmentation and Radial-Crack Zones. Calculations were performed in terms of the initial set of parameters with variation of them in order to reveal the characteristic features of the process and to arrange the parameters, depending on the degree of their effect on the final result for fracture.

Figure 2 shows the dependences $a(t)$ and $b(t)$ (curves 1 and 2), and $l_N(t)$ (curves 3–6 for $N = 4$, 8, 16, and 32) for the initial set of parameters. The estimates of the final value of l_N obtained in this calculation differ by not greater than 5% from those obtained using formulas (23) with the a_k and b_k values from the dynamic calculation and is $l_8 = 276$ and $l_4 = 406$, respectively. The diagrams in Fig. 2 are characterized by a monotone increase in the parameters which is more rapid when the fragmentation zone grows and slower when the radial-crack zone increases. The behavior of the solution changed as soon as the return motion of the explosion cavity begins for small values of internal friction ($\alpha < 1$). For $\alpha = 0.1$ and $Y = 8 \cdot 10^5$ Pa, the dependences $a(t)$, $l_8(t)$, and $\dot{l}_8(t)$ are shown in Fig. 3 (curves 1, 3, and 5, respectively). Variations in the cavity radius during the development of the radial-crack zone lead to the stoppage of the periodic crack and of the formation of subsequent monotone sections of crack elongation. The final length l_8 is in good agreement with the estimate according to formula (23). The sections of curves 2 and 4 in Fig. 3 correspond to $b(t)$ and $\dot{b}(t)$ at the first stage of fracture.

In the calculations, the values of the initial set of parameters were varied as follows: $E = 10^9$–10^{11} Pa, $\rho = 1500$–3500 kg/m^3, $Y = 8 \cdot 10^5$–$800 \cdot 10^5$ Pa, $\alpha = 0.1$–11.2, $\gamma_0 = 50$–200 J/m^2, $v_{\max} = 300$–1300 m/sec,

TABLE 1

P, 10^5 Pa	$Y_2/Y=1$			$Y_2/Y=10$			$Y_2/Y=100$		
	$Y_2 = 8\cdot 10^7$ Pa								
	a	b	l	a	b	l	a	b	l
1	2.4	49.6	106	3.62	73.1	228	3.95	78.2	267
10	2.39	46.6	74.8	3.43	66.1	130	3.68	70.0	144
100	2.25	30.5	31.4	2.63	36.8	38.8	2.71	37.7	39.9
500	1.9	13.4	13.4	1.98	14.1	14.1	1.98	14.2	14.2
	$Y_2 = 8\cdot 10^8$ Pa								
1	1.64	9.15	131	2.22	13.0	276	2.37	13.8	320
10	1.64	9.08	75.9	2.21	12.9	137	2.35	13.6	153
100	1.63	8.46	24.1	2.1	11.8	36.5	2.21	12.1	39.3
500	1.58	6.54	8.48	1.84	8.0	12.4	1.88	8.21	12.9

$P = 10^5$–10^7 Pa, $a_0 = 0.0225$–$0,1$ m, $Y_2 = 8\cdot 10^7$–$8\cdot 10^8$ Pa, and $\alpha_2 = 4$–11.2. The above-considered range of parameters includes the basic rocks (from coal to strong granite and diabases). For the dimensions of the radial-crack zone ($N = 8$), variation in one of the parameters yields the estimate of the effect of various parameters from the initial set:

$$\frac{l}{a_0} \approx \frac{\rho^{0.1} v_{\max}^{0.2} Y_2^{0.5} a_0^{0.5}}{N^{0.5} E^{0.1} Y^{0.1} \alpha^{0.1} \gamma_0^{0.5} P^{0.5} \alpha_2^{0.16}}. \tag{25}$$

Evidently, this dependence indicates the great importance of the parameters Y_2, γ_0, N, a_0, $v_{\max}$, and P for determination of the fragmentation in the radial-crack zone that is caused by an explosion of a cylindrical charge. The more detailed calculations were made with variation of the strength parameters of the medium Y, α, Y_2, and α_2 on the basis of the initial set.

The parameters Y and α, which are the quantities that determine the deformation of the medium on the beyond-the-bounds descending compression branch of the compression diagram, have been poorly studied. Of interest is the effect of the loading rate on the values of the strength parameters Y_2 and α_2. In the initial set of parameters, Y_2 was set larger by approximately a factor of 5 than the average σ_c for rocks and $Y = 0.1Y_2$. Table 1 gives the values of the maximum cavity (a) and fragmentation-zone (b) radii and of the crack length (l) for $N = 8$ as a function of Y_2/Y for the external pressure P with $\alpha = 4$ and $\alpha_2 = 11.2$.

Table 1 shows that a proportional increase in Y_2 and Y leads to a marked decrease in the cavity radius and in the fragmentation zone along with a simultaneous slow increase in the radial-crack zone. Variations in Y_2/Y_1 and P exert a stronger effect on the dimension of the radial-crack zone. For large values of the external pressure, the radial-crack zone decreases compared with the fragmentation zone and can disappear altogether.

Figure 4 shows the dependences a/a_0 and b/a_0 (curves 1 and 2), and l_8/a_0 on P for $a_0 = 0.0225$, 0.05, 0.1, and 0.15 (curves 3–6) and $\gamma_0 = 150$ J/m^2. For other values of γ_0, a_0, and N, the dimensions of the radial-crack zone can be calculated by (23) with a_k and b_k, which are known, for example, from Table 1. We can also find such fracture parameters in this zone as $N_{\max}(l)$ and $l_N = l_{\max}(N)$. For strength parameters different from those indicated in Table 1, it suffices to use one value of N, a_0, and γ_0 from the original set to determine the fracture intensity in the radial-crack zone.

Comparison with the Data of Quasi-Static Models. The proposed description of brittle-medium fracture by the explosion of a cylindrical charge also admits a quasi-static interpretation to establish the final fracture parameters [1, 3]. To this end, it is sufficient to use the static solution in the grinding zone and to adopt the assumption that the medium in the grinding and radial-crack zones is in a limiting state. Here, in (17), $\sigma_1 = \sigma_c$ and instead of (4) we have

$$\sigma_r(r) = Y/\alpha + F r^{-\alpha/(1+\alpha)}. \tag{26}$$

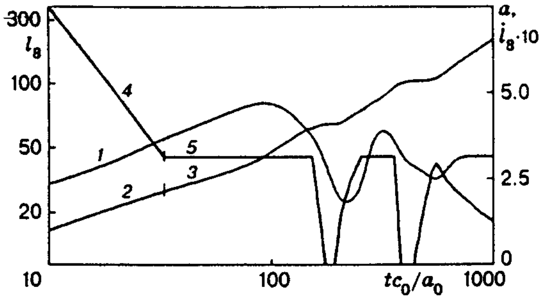

Fig. 3

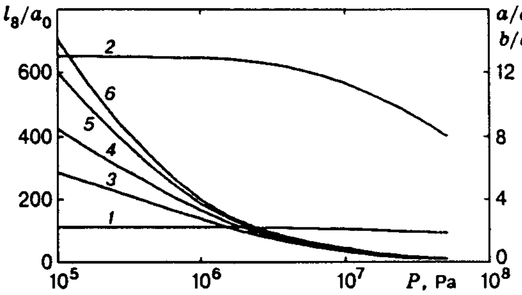

Fig. 4

Applying the general solutions to the grinding and elasticity zones along with the boundary conditions in the cavity (8) and in the fragmentation wave (9), for the desired a and b, in the case of the absence of the radial-crack zone we obtain the following system of algebraic equations:

$$(Y/\alpha - q)(b/a)^{\alpha/(1+\alpha)} - Y/\alpha - p(a) = 0, \quad a^2 - a_0^2 = b^2 - (b - u_b)^2,$$
$$u_b = -\frac{(1+\nu)b(q+P)}{E}, \qquad q = -\frac{Y_2 + 2(1+\alpha_2)P}{2+\alpha_2}. \tag{27}$$

The limiting equilibrium state of the grinding, radial-crack, and elasticity zones is defined by the relations

$$(Y/\alpha + \sigma_c)(b/a)^{\alpha/(1+\alpha)} - Y/\alpha - p(a) = 0, \qquad a^2 - a_0^2 = b^2 - (b - u_b)^2,$$
$$u_b = -\frac{(1+\nu)b(q+P)}{E} + \frac{(1-\nu^2)\sigma_c}{E} b \ln \frac{l}{b}, \quad q = -\sigma_c(b/l), \quad N = \frac{\pi l(1-\nu^2)}{2\gamma_0 E}(q + 2P)^2, \tag{28}$$

which are derived from the general solutions (6), (7), and (27) and the boundary conditions (8) and (17).

For the initial set of parameters, the calculation by formulas (27) yields $a = 1.5$ and $b = 8.3$, which are markedly smaller than the results of the dynamic calculations ($a_k = 2.2$ and $b_k = 13$ for the conservative values of the cavity and grinding-zone radii, see Fig. 2).

The static estimates of the crack size are considerably larger than the dynamic ones: $l_8 \approx 667$ instead of 276 in dynamics. Here the static solution in the presence of the grinding, radial-crack, and elasticity zones does not exist, and an estimate was made using the scheme of two zones: a zone of radial cracks which originate in the cavity and an elasticity zone. In this variant, the solution for a and l is found from the system of equations in dimensionless form:

$$p(a) = \frac{a - 1 + (1+\nu)lN}{(1+\nu)[(1-\nu)\ln l + 1]}, \qquad N = \frac{\pi l(1-\nu^2)}{2\gamma_0}\left(p(a)\frac{a}{l} - 2P\right)^2, \tag{29}$$

which were obtained from solutions (6) and (7) and conditions (8) and (17).

The overestimated crack dimensions which were obtained by the static calculations are due to the underestimated dimensions of the grinding zone. In the dynamic calculation, the grinding zone is found with allowance for fragmentation-wave propagation and inertia of the medium. In this case, large values for b gives rise to decreasing l, because the compliance of the medium in the grinding zone is larger than in the radial-crack zone. Owing to this, with cracks in an equilibrium state, the pressure in the cavity proves to be somewhat smaller for large b, thus leading to the reduction of l.

Basic Results.

(1) We have designed a scheme of calculation of the fracture parameters for a brittle medium which is caused by the explosion of a cylindrical charge with allowance for the dynamics of the grinding zone and crack propagation.

(2) Throwing the material in the grinding zone deep in the medium at the stage of its development and blocking the material upon its inverse compression have been shown to be the determining factors for the

development of the radial-crack zone.

(3) Various parameters have been ranked based on their effect on the result of the explosion, and the most important ones have been selected, namely, σ_c, γ_0, $v_{\max}$, Y, P, and a_0. In this connection, the problem arises of the determination, for rocks, of the dynamic uniaxial-compression parameters σ_c and yielding γ_0, γ_1, and $v_{\max}$ and also of the cohesion and friction coefficients at the grinding zone.

(4) The calculation results in the dynamic and static approximations have been compared. We have shown that according to static models, the cavity and grinding-zone dimensions in the explosion of a cylindrical charge turn out to be underestimated, and the dimension of the radial-crack zone turns out to be grossly overestimated, which shows the necessity of taking into account the dynamics.

(5) In the present work, the processes of detonation-product flow along a well and their penetration into the grinding zone have not been taken into account and need further investigation.

This work was supported by the Russian Foundation for Fundamental Research (Grant No. 95-01-01169).

REFERENCES

1. P. A. Martynyuk and E. N. Sher, "Estimation of the dimensions of a radial-crack zone formed in a contained camouflet explosion of a line charge in a brittle medium," *Zh. Prikl. Mekh. Tekh. Fiz.*, **25**, No. 4, 127–132 (1984).
2. E. N. Sher, "Dynamic development of a radial-crack zone during a camouflet explosion," *Zh. Prikl. Mekh. Tekh. Fiz.*, **29**, No. 1, 164–167 (1988).
3. V. N. Rodionov, V. V. Adushkin, A. N. Romashev, et al., *Mechanical Effects of an Underground Explosion* [in Russian], Nedra, Moscow (1971).
4. S. S. Grigor'yan, "Some questions of the mathematical theory of deformation and failure of solid rocks," *Prikl. Mat. Mekh.*, **31**, No. 4, 643–649 (1967).
5. G. P. Cherepanov, *Mechanics of Brittle Fracture* [in Russian], Nauka, Moscow (1974).
6. P. Chadwick, A. Cox, and H. Hopkins, "Mechanics of deep underground explosions," *Philos. Trans. Roy. Soc. London, Ser. A*, **256**, No. 1070 (1964).
7. V. I. Mashukov, V. D. Baryshnikova, and N. V. Pirlya, "Rock structure and its strength certificate," *Fiz. Tekh. Probl. Razrab. Polezn. Iskop.*, No. 3, 21–27 (1990).
8. A. N. Stavrogin and A. G. Protosenya, *Strength of Rocks and Shaft Stability at Large Depths* [in Russian], Nedra, Moscow (1985).
9. E. N. Sher, "Example of calculation of the motion of radial cracks formed by an explosion in a brittle medium in the quasi-static approximation," *Fiz. Tekh. Probl. Razrab. Polezn. Iskop.*, No. 2, 40–42 (1982).
10. E. N. Sher, "Equilibrium of a radial-crack system," *Zh. Prikl. Mekh. Tekh. Fiz.*, No. 5, 156–159 (1974).
11. R. A. Westman, "Pressurized star crack," *J. Math. Phys.*, **43**, No. 3, 191–198 (1964).